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APPLICATION OF Z-NUMBER THEORY TO UNCERTAINTY MODELING IN ENERGY CONSUMPTION SYSTEMS

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ПРИМЕНЕНИЕ ТЕОРИИ Z-ЧИСЕЛ ДЛЯ МОДЕЛИРОВАНИЯ НЕОПРЕДЕЛЁННОСТИ В СИСТЕМАХ ЭНЕРГОПОТРЕБЛЕНИЯ

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Abstract. The article examines the application of Z-number theory in modeling uncertainty in energy consumption systems. It is shown that since modern energy systems are dynamic and multifactorial in nature, uncertainty arises from various sources in the process of forecasting and managing consumption. Although traditional probabilistic-statistical models take into account the random nature of information, they remain limited in providing a complete mathematical description of information with different levels of imprecision and reliability. The Z-number approach allows for the formal modeling of double uncertainty by combining two main characteristics of information within the same model. The study explains the mechanism of modeling energy consumption based on Z-numbers and shows its application possibilities in the fields of peak load forecasting, energy distribution optimization, and risk assessment in smart grid systems. Consequently, the application of Z-number theory creates a scientific and methodological basis for a more realistic and justified decision-making process in energy systems, a more accurate assessment of risks, and an increase in system stability.

Аннотация. Рассматривается применение теории Z-чисел для моделирования неопределённости в системах энергопотребления. Показано, что современные энергетические системы имеют динамический и многофакторный характер, вследствие чего в процессах прогнозирования и управления энергопотреблением возникает неопределённость, обусловленная различными источниками. Несмотря на то, что традиционные вероятностно-статистические модели учитывают случайный характер информации, они остаются ограниченными при математическом описании данных, характеризующихся различной степенью нечёткости и надёжности. Подход, основанный на Z-числах, позволяет формально моделировать двойственную неопределённость, объединяя две основные характеристики информации в рамках единой модели. В исследовании раскрыт механизм моделирования энергопотребления на основе Z-чисел, а также показаны возможности его применения в прогнозировании пиковых нагрузок, оптимизации распределения энергии и оценке рисков в интеллектуальных энергетических сетях. В результате установлено, что применение теории Z-чисел формирует научно-методологическую основу для более реалистичного и обоснованного процесса принятия решений в энергетических системах, более точной оценки рисков и повышения устойчивости функционирования системы.

Keywords: energy consumption, smart grid, anomaly detection, soft computing, neural networks, hybrid models.

Ключевые слова: энергопотребление, интеллектуальные энергетические сети (Smart Grid), обнаружение аномалий, мягкие вычисления (Soft Computing), нейронные сети, гибридные модели.

Introduction. In modern times, the energy system acts as one of the main pillars of economic development, industrial production, and social welfare. Population growth, the acceleration of urbanization processes, the wide application of digital technologies, and the spread of electric vehicles have led to a continuous increase in energy consumption. This necessitates more flexible, sustainable, and intelligent management of energy systems. In particular, the increase in the share of renewable energy sources in the energy balance has led to a more complex and nonlinear nature of the relationship between energy production and consumption. Uncertainty in energy consumption systems is associated with numerous factors. Climate variability, the difficulty of predicting consumer behavior, market price volatility, measurement errors, and data gaps complicate the decision-making process [9].

While traditional statistical and probabilistic models are effective in certain situations, the presence of imprecise, language-based, and subjective expert data in real energy systems exposes the limitations of these approaches. In this regard, fuzzy logic and alternative models of uncertainty are of particular relevance. One of the important directions in modeling uncertainty is the Z-number theory proposed by Lotfi A. Zadeh. The Z-number provides not only a description of fuzzy information but also a mathematical formalization of the level of reliability of that information. This feature allows for more realistic and adaptive construction of decision-making mechanisms in dynamic and multifactor environments, such as energy consumption systems [1].

The Z-number model is based on taking into account double uncertainty. This approach provides more flexible and practical solutions in issues such as energy consumption forecasting, peak load management, risk assessment, and resource optimization. Especially in the context of smart energy networks (smart grids), the inaccuracy of real-time data and the different reliability levels of information coming from different sources make the application of the Z-number theory even more relevant.

The purpose of the study is to investigate the theoretical and methodological foundations of modeling the uncertainties existing in energy consumption systems based on the Z-number theory, to determine the advantages of this approach, and to analyze its application possibilities in decision-making processes. Research in this direction can contribute to increasing the resilience and forecasting accuracy of energy systems.

Main part

Characteristics of uncertainty in energy consumption systems. Energy consumption systems are socio-technical systems with a complex, multi-level, and dynamic structure. These systems cover the stages of energy production, transmission, distribution, and final consumption, and are formed as a result of the interaction of various technological, economic, and social factors. The continuity of energy flows and the stability of the system depend on the synchronous activity of numerous variables. In this regard, uncertainty acts as a natural and inevitable feature in energy consumption systems [8].

Uncertainty arises under the influence of both internal and external factors. Among the internal factors, the variability of consumer behavior should be noted first of all. The energy use of the

population and enterprises varies under the influence of daily, seasonal, and even psychological factors. For example, a sudden decrease in temperature can lead to a sharp increase in electricity and gas consumption. At the same time, the production volume, work schedule, and level of technological modernization of industrial enterprises create serious changes in energy demand. Technical failures, equipment wear and tear, and network losses are also important factors that increase uncertainty within the system. The inaccuracy of measuring devices and delays in data transmission make real-time decision-making difficult.

External factors cover a wider spectrum. Climate and meteorological variability are among the main determinants of energy consumption. In particular, the increase in the share of renewable energy sources (wind, solar, etc.) in the energy balance causes the balance between production and consumption to become more sensitive. The rate of economic growth, the level of inflation, and the development of the industrial and service sectors directly affect energy demand. In addition, price changes in energy markets, international energy policy, and geopolitical risks are also important sources of uncertainty affecting the behavior of the system.

The nature of uncertainty in energy consumption systems is multidimensional and nonlinear. There are interdependencies and feedback mechanisms between variables. For example, an increase in energy prices can lead to a decrease in consumption, but this effect manifests itself differently across social and economic groups. At the same time, the introduction of information technologies and digital management systems creates new information flows, which leads to both a decrease in uncertainty and the emergence of new forms of uncertainty.

Traditional probabilistic-statistical models model this uncertainty mainly through random variables and distributions. However, in real energy systems, information is often inaccurate, incomplete, or qualitative. For example, statements such as “high demand,” “moderate risk,” “relatively stable dynamics,” or “sufficiently reliable forecast” given by experts cannot be directly formalized in classical mathematical models. Although such language-based and subjective assessments are widely used in energy management practice, their mathematical modeling requires a special methodological approach.

In this regard, fuzzy logic and its development, the Z-number theory, allow for a more adequate description of uncertainty in energy consumption systems. The fuzzy approach provides for the expression of variables not by rigid boundaries, but by membership degrees. In addition, the Z-number models double uncertainty, taking into account the level of reliability of the information provided. Uncertainty in energy consumption systems is not limited to statistical randomness; there are also problems of inaccuracy, incompleteness, subjectivity and reliability. These features necessitate the analysis of modern energy systems with more flexible and intelligent models and scientifically justify the application of alternative mathematical approaches, including the Z-number theory.

Theoretical and methodological foundations of the Z-number theory. The concept of Z-number was put forward at the stage of development of the fuzzy logic theory and expanded its more flexible, more realistic modeling capabilities. This concept was proposed by Lotfi A. Zadeh and aimed at a more complete description of uncertain information. While the classical fuzzy number only reflected the imprecise nature of the variable, the Z-number takes this description a step further and also takes into account the level of reliability of the information [1].

A Z-number can be presented as follows: $Z=(A,B)$. Here: A is a fuzzy constraint of the variable. This component expresses an imprecise description of a certain event or indicator. For example, the concept of “high energy consumption” is characterized not by a specific number, but by a certain membership function.

B is a second fuzzy function indicating the degree of reliability of that constraint. For example, assessments such as “high reliability” or “medium reliability” are presented in mathematical form through this component.

The Z-number structure combines two important properties of information: the first is the imprecision of the variable, and the second is the degree of reliability of that information. In real systems, especially in complex and variable areas such as energy consumption, these two factors are inseparable. Because often information is not only imprecise, but also has different levels of reliability.

From a methodological point of view, the Z-number approach allows for formal modeling of expert assessments based on language. Decision-making processes in the energy sector are often based on statements such as “demand may be high,” “risk is moderate,” and “forecast is reasonably accurate.” Z-number theory provides a mathematical interpretation of such statements and enables their integration into computational models.

In addition, the Z-number concept enables a mathematical description of double uncertainty:

Inaccuracy of information – the fact that the variable is expressed not by hard boundaries, but by membership degrees;

Reliability of information – an additional function that indicates how reliable this estimate is.

This approach is more flexible than classical probability and statistical models, because it can combine both quantitative and qualitative information within a single model.

Z-number theory creates a methodological basis for more realistic and justified decision-making in complex energy systems.

Z-number-based modeling of energy consumption.

The main goal when modeling energy consumption is to build a more adequate mathematical description of the changing and uncertain demand. Traditional models are often based on specific numbers or probability distributions. However, in real conditions, information about energy consumption is both imprecise and has different levels of reliability. Therefore, the Z-number approach provides a more flexible and practical model.

The Z-number for the energy consumption variable X is expressed in the following form: $Z_X = (A_X, B_X)$.

Where: A_X – is a fuzzy set expressing the level of energy consumption (for example, low, medium, high, etc.). This component ensures that consumption is characterized not by an exact number, but by a certain interval and a degree of membership. B_X – is the reliability function of the forecast or measurement. This function shows how reliable the given assessment is.

For example, an expert gives such an assessment: “Tomorrow electricity consumption will be high and the forecast is highly reliable.”

This statement can be modeled with a Z-number structure as follows: A_X – is the membership function built for “high consumption” (for example, an increasing degree of membership in a certain kWh interval); B_X – is the membership function for “high reliability” (for example, a high level of confidence in the interval 0.7–1).

The model takes into account both the level of consumption and the degree of reliability of that forecast simultaneously. This allows for a more realistic assessment of risks in the decision-making process [4].

Model application stages:

Z-number modeling of energy consumption consists of several sequential stages:

1. *Definition of linguistic variables.* Terms such as “low”, “medium”, “high”, “peak level” are defined for energy consumption.

2. *Construction of membership functions.* An appropriate mathematical function (for example, a triangular or trapezoidal function) is defined for each linguistic variable.

3. *Modeling of the reliability function.*

The reliability of the forecast given by the expert or model is constructed as a separate fuzzy function.

4. *Application of Z-number operations.* Operations such as comparison, summation or optimization are performed taking into account the two-component structure.

5. *Formation of a decision rule.* For instance, in the case of high consumption and high reliability, a management decision can be made such as activating additional generation capacity.

This approach provides effective results, especially in peak load management, energy resource planning and integration of renewable energy sources. Thanks to the Z-number model, not only the expected consumption level, but also the risk level of that forecast is taken into account. Z-number-based modeling of energy consumption allows for more realistic, flexible, and reliable decision-making processes.

Application possibilities in smart grid systems. Smart grids represent the digital and intellectual stage of modern energy systems. These systems collect and process large amounts of data in real time and manage energy flows through automated decision-making mechanisms. Sensor networks, smart meters, distributed generation sources and control algorithms are combined into a single information platform, increasing the efficiency and stability of the system [7].

However, the intensity of the information flow in smart grid systems does not mean the complete elimination of uncertainty. On the contrary, the level of accuracy of the information received from different sources is different. Technical errors in sensor measurements, delays in data transmission, variability of meteorological forecasts and the fact that models give different results complicate the decision-making process. In addition, the reliability level of different forecasting algorithms is not the same, and this factor directly affects the optimization of the system. In this context, the Z-number approach acts as an effective mathematical tool for more adequate modeling of uncertainty in smart grid systems. The Z-number structure allows us to simultaneously consider both the quantitative side of information (e.g., energy level) and the qualitative side (reliability level).

The main application areas of the Z-number approach in smart grid systems are as follows:

1. *Peak load forecasting.* Accurately forecasting peak energy demand levels is critical for system stability. Assessments such as “high peak probability” and “medium reliability” are formalized through Z-numbers, allowing for more informed decisions on additional generation or activation of reserve capacities.

2. *Energy distribution optimization.* During the optimal distribution of energy flows within the network, both the consumption level and the reliability of forecasts should be taken into account. The Z-number model combines these two factors, allowing for a more balanced distribution strategy.

3. *Managing the variability of renewable energy sources.* Resources such as wind and solar energy have natural variability. For example, when assessments such as “medium power” and “medium reliability” for wind energy are modeled with a Z-number structure, the system can automatically activate alternative energy sources or backup mechanisms. This ensures the preservation of the energy balance.

4. *Risk assessment.* The probability of a network failure, the risk of overloading, or energy shortages should be assessed not only in terms of the probability, but also in terms of the reliability of that probability. The Z-number makes risk analysis more realistic and multidimensional. The application of Z-number theory in smart grid systems enhances adaptive and intelligent management. The system makes decisions taking into account not only the expected indicators, but also the level

of reliability of these indicators. This serves to increase energy security, more efficient use of resources, and generally strengthen the resilience of energy systems.

Advantages and limitations of the Z-number approach. Z-number theory provides a more flexible and multidimensional method for modeling uncertainty compared to traditional mathematical approaches. Especially in complex and dynamic environments such as energy consumption systems, the application of this approach is characterized by a number of important advantages. However, there are also certain difficulties in the practical application of the method [7].

Advantages:

1. Taking into account double uncertainty. The main advantage of the Z-number is that it models not only the uncertainty of information, but also the level of reliability of that information. This feature allows for a more objective assessment of risks and forecasts in energy systems.

2. Formal modeling of expert knowledge. Decision-making in the energy sector is often based on expert opinions. Expressions such as “high probability of demand”, “medium risk level” are difficult to formalize in classical statistical models. The Z-number theory allows such qualitative information to be put into mathematical form and included in the calculation process.

3. Mathematical expression of language-based data. In real systems, data is not always in a precise numerical form. The integration of fuzzy and language-based variables into mathematical models provides a more flexible and practical structure of the decision-making process.

4. More realistic structure of the decision-making process. Since the Z-number also takes into account the degree of reliability of forecasts and estimates, the decisions made are more justified and balanced in terms of risk. This feature plays an important role, especially in smart grid systems and in the management of renewable energy sources.

Limitations:

1. Complexity of the computational process. Since the Z-number structure consists of two components, the calculations are more complex than in classical fuzzy models. In large-scale systems, this may require additional computational resources.

2. Subjectivity in the selection of membership functions. The selection of membership functions for both the main variable and the reliability component is based on an expert approach to some extent. This may create differences in the interpretation of model results.

3. Integration difficulties in a big data environment. Modern energy systems work with large amounts of information. The integration of the Z-number approach into big data platforms in real time requires additional algorithmic and software solutions from a technical point of view.

The role of the Z-number approach in soft computing systems. The concept of soft computing is a set of methods aimed at building effective decision-making and computational processes in an environment of imprecise, uncertain and incomplete information. This approach, unlike classical hard (hard computing) mathematical models, is based on the principles of flexibility, tolerance and adaptability. The soft computing framework includes fuzzy logic, artificial neural networks, genetic algorithms, evolutionary computation, and other intelligent methods. The heterogeneity and uncertain nature of data in modern energy systems makes the application of this approach particularly relevant.

Z-number theory acts as an important methodological element in soft computing architecture. While fuzzy logic provides an imprecise description of variables, Z-number makes modeling deeper and multidimensional by taking into account the level of reliability of that description. This feature is of particular importance in structuring data coming from different sources in energy consumption systems. For example, when sensor measurements, meteorological forecasts, and expert opinions are used simultaneously, their reliability levels may be different. The Z-number approach allows these differences to be expressed formally [5].

The application of Z-number in soft computing systems manifests itself in several ways. First of all, in the process of structuring imprecise information, Z-number models both the content and reliability of information, thereby creating conditions for more stable operation of decision mechanisms. Second, the Z-number approach plays an important role in improving expert systems. Since decisions made in the energy sector are often based on language-based assessments, the transformation of this information into a mathematical form increases the effectiveness of expert systems. In addition, the Z-number concept, together with neural networks and genetic algorithms, creates a methodological basis for building hybrid optimization models. For example, a neural network can learn and predict energy consumption patterns, a genetic algorithm can select optimal parameters, and Z-number can provide a risk analysis of the result, taking into account the level of reliability of these forecasts and parameters. Such hybrid models form an effective mechanism for adaptive control and anomaly detection, especially in smart grid systems. The Z-number approach acts not only as an additional mathematical tool within the framework of soft computing, but also as an integrative mechanism that provides multidimensional modeling of uncertainty. This approach creates a scientific and methodological basis for building a more flexible, adaptive and reliable decision-making process in energy consumption systems.

Conclusion

The analysis shows that since energy consumption systems are complex, dynamic and multifactorial in nature, modeling uncertainty in this area requires a special scientific approach. Climate variability, instability of consumer behavior, technical errors and market factors complicate the decision-making process in energy systems. Although traditional statistical and probabilistic models are effective in certain cases, they have limitations in providing a complete description of data with imprecision and different levels of reliability. The Z-number theory allows for the consideration of double uncertainty within the same model. This approach ensures the achievement of more realistic and justified results in issues such as energy consumption forecasting, peak load management, risk assessment and resource optimization. Especially in smart grid systems, the different levels of reliability of data coming from different sources make the application of the Z-number model even more relevant. The research results show that the Z-number approach has significant advantages in terms of formalization of expert knowledge and mathematical modeling of language-based data. However, limitations such as computational complexity and subjectivity in the selection of membership functions should be taken into account. Overall, the application of Z-number theory to energy consumption systems not only increases the accuracy and reliability of decision-making mechanisms, but also contributes to strengthening the system's resilience. Future research in this direction can explore the integration of Z-number models with big data and artificial intelligence algorithms, thereby creating a scientific basis for more efficient and adaptive management of energy systems.

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